Ancient TL

Luminescence as a Relative Dating Tool: Part A – Theory

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Received: June 14, 2019; in final form: December 12, 2019

Abstract

A formula is derived for calculating relative numerical ages of ceramic vessels using optically stimulated luminescence (OSL). These relative numerical ages may be generated when the standard absolute numerical ages cannot be determined; this is usually because a component of the dose rate such as the external dose rate cannot be measured or deduced. The error associated with this relative age formula is derived. It is shown that, where external dose rate information is unavailable, the error in the relative age that results from this lack of information is much smaller than the equivalent error in the absolute age.

Keywords: OSL dating, relative dating, museum material, ceramics, archaeology

1. Introduction

An undisputed accomplishment of luminescence dating has been the calculation of numerical ages for archaeological assemblages, most frequently based upon ceramic or sediment dating. Indeed, the application of luminescence dating has significantly impacted how we look at, and understand, past human activity. Yet luminescence can also be used as a *relative* numerical dating technique, whence it is possible to establish typological and chronological sequences within archaeological material and create a floating, relative typological framework for the material studied.

In general, numerical dating (whether OSL, radiocarbon, or other) produces an *absolute* age; that is, it results in an age which is a known number of years before the present day, and which allows the associated event (e.g. use/manufacture) to be assigned a calendar date. By contrast, relative dating (e.g. stratigraphy) does not produce a numerical age (absolute or otherwise), unless it is linked to another chronology, whether derived by historical or scientific methods. In this paper we discuss the creation of a *relative numerical* chronology. Like a traditional relative chronology, this numerical relative chronology allows us to assign a sequence to ancient events, without placing them on a calendar. Like a standard numerical chronology, we calculate a numerical age for each sample; this numerical age does not measure the number of years before present, but it does allow the formulation of statements like "this sample is twice as old as that one". Throughout this paper, we will refer to numerical methods that produce a calendar age (radiocarbon, standard OSL) as "absolute" methods, and to any methods (numerical or otherwise) that do not produce a calendar age as "relative" methods. To distinguish between the relative numerical method presented here and methods such as stratigraphy, we refer to the latter as "non-numerical relative dating" or "traditional relative dating" methods.

Previously, relative OSL dating has not been carried out for two reasons: first, in cases where access to recently excavated field material is forthcoming for luminescence dating, using the technique as a relative dating method is not necessary as the additional components required to calculate an absolute age will be available. Secondly, owing to the associated costs of luminescence analysis as a relative dating technique, alternative relative dating techniques, such as seriation, would be the more accepted option in many cases. However, there are clear areas of research where using luminescence as a relative dating tool would be of benefit to the archaeologist: for example, ceramic assemblages in museums, whose provenance and chronology are uncertain (particularly when used in combination with the minimum extraction technique (MET) which is a method for obtaining optically stimulated luminescence (OSL) dates from museum materials; Hood & Schwenninger, 2015). For such material, a set of relative luminescence dates might well provide significant new insights, and perhaps even be the only robust way of establishing their relative chronology. The reasons for needing to rely upon museum material, rather than recently excavated material are varied, but include political and geographical disruptions which render access to an original excavation site impossible.

Many museum pieces were acquired in an era when detailed recording of provenance and archaeological context was rare, often with specimens being bought from antiquities dealers with no indication of the true find-spot of a piece, other than a broad regional location. Often this would happen to entire classes of vessels, those that were the more prized, probably owing to a particular characteristic of the ware, and in turn significant information was lost. A particular case in point is the Predynastic Egyptian Decorated Ware, or D-Ware, one of nine wares classified by Petrie in his Corpus of Prehistoric Pottery (Petrie, 1921). Although Petrie (whose recording methods were arguably the best of his generation) along with his peers excavated a large number of such vessels for museum collections, a large number were also acquired by museums through other means (e.g purchase) to add to their collections, and often without verified provenance.

Without their origin being known (and having been well cleaned prior to display), these vessels lack a crucial piece of information for determining an absolute luminescence date: the external dose rate ($\dot{D}_{ext}$), which can be obtained using original sediment adhering to the vessel. Without the $\dot{D}_{ext}$ measurement, only a relative date can be achieved by taking the equivalent dose (D_e) and dividing it by the internal dose rate ($\dot{D}_{int}$), rather than dividing it by the sum of $\dot{D}_{ext}$ and $\dot{D}_{int}$. While the lack of information about $\dot{D}_{ext}$ will mean that this relative date can deviate significantly from the true absolute age of the pot, the relative date may be used to determine the relative sequence of the ceramics in a manner similar to traditional relative dating techniques such as seriation, that is, these dates will help determine the sequence of the vessels. This relative date would depend on the assumptions that $\dot{D}_{ext}$ is both small compared to $\dot{D}_{int}$ (often the case for ceramic material) and similar for all the ceramics being studied. The second assumption may be justified on a variety of grounds, for example the ceramics come from one context; the ceramics come from contexts constructed from common building materials; the ceramics come from contexts with the same geology; the ceramics comes from a region where the natural background radiation has been measured across that region and shown to have little variation. The merit of this assumption must be argued on a case-by-case basis; in this paper we study a group of ceramics from several similar (radiometrically speaking) contexts at a single site.

This paper is accompanied by a second paper (Hood et al., 2019), which we refer to as Part B. This work, Part A, presents a derivation of the formulae for obtaining both a relative luminescence age and the associated relative error. Part B, which follows directly, presents a case study on determining the relative age using OSL dating, carried out on a group of ancient Egyptian ceramics.

2. Formal derivation of a relative age formula and associated error

In order to accurately apply luminescence as a relative dating technique, it is essential to determine how to calculate the approximate relative age of vessels using only the D_e and $\dot{D}_{int}$ measurements, as well as how to construct an estimate of the error associated with this calculation, which results from the fact that $\dot{D}_{ext}$ is neglected.

To derive the approximation and associated error, we must start from a mathematical expression for the relative age. To formulate such an expression we temporarily assume knowledge of all the parameters, including the external dose rate, that are required to derive hypothetical (absolute) numerical ages for each vessel. We then define the relative age of two vessels to be the hypothetical (absolute) numerical age of one divided by the hypothetical (absolute) numerical age of the other.

Having defined this relative age, we make an approximation of it by using asymptotic theory, and identifying certain parameters which we expect to be small (we will estimate them and verify that they are small subsequently). In common with standard asymptotic approaches, we wish to derive an approximation that tends to the original expression in the limit that the parameters become infinitely small. When (as is typically the case) the parameters are finite, there is an error associated with the approximation whose size we can estimate.

Mathematically, the relative age, R, of two vessels is defined as:

$$R \equiv \frac{A_1}{A_2}, \tag{1}$$

where A_1 and A_2 are the hypothetical (absolute) numerical ages of the two individual vessels respectively. Now,

$$A_1 = \frac{D_{e,1}}{\dot{D}_{int,1} + \dot{D}_{ext,1}} \tag{2}$$

and

$$A_2 = \frac{D_{e,2}}{\dot{D}_{int,2} + \dot{D}_{ext,2}}, \tag{3}$$

where $D_{e,1}$, $\dot{D}_{int,1}$ and $\dot{D}_{ext,1}$ are the equivalent dose, the internal dose rate and the external dose rate, respectively, for the first vessel and $D_{e,2}$, $\dot{D}_{int,2}$ and $\dot{D}_{ext,2}$ are the same measurements for the second vessel. Therefore,

$$\frac{A_1}{A_2} = \frac{D_{e,1}}{D_{e,2}} \frac{\dot{D}_{int,2} + \dot{D}_{ext,2}}{\dot{D}_{int,1} + \dot{D}_{ext,1}}. \tag{4}$$

If $\dot{D}_{ext,1}$ and $\dot{D}_{ext,2}$ (i.e. $\dot{D}_{ext}$) were known, we would know the exact relative age of the two vessels. However, even though these values are unknown, it is possible to calculate an approximate relative age for both of the vessels, and, additionally, an estimation of the error associated with that approximation. There are three assumptions required: first, that $\dot{D}_{int}$ is similar for both vessels; secondly, that $\dot{D}_{ext}$ is similar for both vessels; thirdly, that for both vessels $\dot{D}_{ext}$ is smaller than $\dot{D}_{int}$.

More formally, we can define three parameters ε_I, ε_E and δ as follows:

$$\varepsilon_I \equiv \dot{D}_{int,2} - \dot{D}_{int,1} \implies \dot{D}_{int,2} = \dot{D}_{int,1} + \varepsilon_I, \tag{5}$$

$$\varepsilon_E \equiv \dot{D}_{ext,2} - \dot{D}_{ext,1} \implies \dot{D}_{ext,2} = \dot{D}_{ext,1} + \varepsilon_E, \tag{6}$$

$$\delta \equiv \frac{\dot{D}_{ext,1}}{\dot{D}_{int,1}} \implies \dot{D}_{ext,1} = \delta \dot{D}_{int,1}, \tag{7}$$

and we further assume that these parameters are small, that is,

$$\left| \frac{\varepsilon_I}{\dot{D}_{int,1}} \right| \ll 1, \left| \frac{\varepsilon_E}{\dot{D}_{ext,1}} \right| \ll 1, \delta \ll 1. \tag{8}$$

The meaning of these parameters, and the justification for assuming that all three are small, will be discussed below.

Now ε_I and ε_E can be substituted directly into the equation for the relative age (4):

$$\frac{A_1}{A_2} = \frac{D_{e,1}}{D_{e,2}} \frac{\dot{D}_{int,1} + \dot{D}_{ext,1} + \varepsilon_I + \varepsilon_E}{\dot{D}_{int,1} + \dot{D}_{ext,1}} \tag{9}$$

$$= \frac{D_{e,1}}{D_{e,2}} \left(1 + \frac{\varepsilon_I + \varepsilon_E}{\dot{D}_{int,1} + \dot{D}_{ext,1}} \right) \tag{10}$$

$$= \frac{D_{e,1}}{D_{e,2}} \left(1 + \frac{\varepsilon_I}{\dot{D}_{int,1} + \dot{D}_{ext,1}} + \frac{\varepsilon_E}{\dot{D}_{int,1} + \dot{D}_{ext,1}} \right). \tag{11}$$

Note that no approximations have been made up to this point, that is, the three assumptions have not yet been utilised.

At this point, it is possible to simply approximate the relative age as $D_{e,1}/D_{e,2}$, in which case the error would be given by the last two terms of equation (11), assuming, of course, that ε_I and ε_E are both small. However, is it possible to improve upon this estimate as follows.

First, the definition of δ is substituted into equation (11):

$$\frac{A_1}{A_2} = \frac{D_{e,1}}{D_{e,2}} \left(1 + \frac{\varepsilon_I}{\dot{D}_{int,1}(1+\delta)} + \frac{\varepsilon_E}{\dot{D}_{int,1}(1+\delta)} \right). \tag{12}$$

The ratio δ is now assumed to be small, which allows the following approximation to be made (using a Taylor series):

$$\frac{1}{1+\delta} \approx 1 - \delta \tag{13}$$

which means that

$$\frac{A_1}{A_2} \approx \frac{D_{e,1}}{D_{e,2}} \left(1 + \frac{\varepsilon_I}{\dot{D}_{int,1}}(1-\delta) + \frac{\varepsilon_E}{\dot{D}_{int,1}}(1-\delta) \right) \tag{14}$$

and therefore

$$\frac{A_1}{A_2} \approx \frac{D_{e,1}}{D_{e,2}} \left(1 + \frac{\varepsilon_I}{\dot{D}_{int,1}} - \frac{\varepsilon_I}{\dot{D}_{int,1}}\delta + \frac{\varepsilon_E}{\dot{D}_{int,1}}(1-\delta) \right). \tag{15}$$

However,

$$1 + \frac{\varepsilon_I}{\dot{D}_{int,1}} = \frac{\dot{D}_{int,2}}{\dot{D}_{int,1}}, \tag{16}$$

so

$$\frac{A_1}{A_2} \approx \frac{D_{e,1}}{D_{e,2}} \left(\frac{\dot{D}_{int,2}}{\dot{D}_{int,1}} - \frac{\varepsilon_I}{\dot{D}_{int,1}}\delta + \frac{\varepsilon_E}{\dot{D}_{int,1}}(1-\delta) \right). \tag{17}$$

It can be seen that the second term in this equation ($\varepsilon_I \delta / \dot{D}_{int,1}$) is second order, being the product of two small parameters. Furthermore, we note also that while δ is positive definite, both ε_I and ε_E can be either positive or negative. Thus, when estimating the error, the second term would have to be added to the third term in quadrature: assuming δ is sufficiently small, we may safely drop this second term, meaning that

$$\frac{A_1}{A_2} \approx \frac{D_{e,1}}{D_{e,2}} \frac{\dot{D}_{int,2}}{\dot{D}_{int,1}} \left(1 + \frac{\varepsilon_E}{\dot{D}_{int,1}} \frac{\dot{D}_{int,1}}{\dot{D}_{int,2}}(1-\delta) \right). \tag{18}$$

Equation (18) demonstrates that the relative age can be approximated by

$$R \equiv \frac{A_1}{A_2} \approx \frac{D_{e,1}}{D_{e,2}} \frac{\dot{D}_{int,2}}{\dot{D}_{int,1}} \tag{19}$$

with a relative error given by:

$$\frac{1}{R} \frac{\varepsilon_E}{\dot{D}_{int,2}}(1-\delta). \tag{20}$$

Rearranging equation (19), it can be seen that

$$R \equiv \frac{A_1}{A_2} \approx \frac{D_{e,1}/\dot{D}_{int,1}}{D_{e,2}/\dot{D}_{int,2}}, \tag{21}$$

and comparing this equation with (4), it can be seen that the approximation of the relative age of two vessels is simply effected by neglecting $\dot{D}_{ext}$, the external dose rate. However, the important point is that the relative error in this relative age is significantly smaller than the relative error in the individual absolute ages that could be calculated by neglecting $\dot{D}_{ext}$.

The relative deviation (which results from neglecting $\dot{D}_{ext}$) in the absolute age (of, for example, the first vessel) is given by

$$\frac{D_{e,1}/\dot{D}_{int,1} - D_{e,1}/\left(\dot{D}_{int,1} + \dot{D}_{ext,1}\right)}{D_{e,1}/\left(\dot{D}_{int,1} + \dot{D}_{ext,1}\right)} \tag{22}$$

$$= \frac{1/\dot{D}_{int,1} - 1/\left(\dot{D}_{int,1}(1+\delta)\right)}{1/\left(\dot{D}_{int,1}(1+\delta)\right)} \tag{23}$$

$$= \delta. \tag{24}$$

3. Estimating the uncertainty of the approximate relative age

The relative deviations in the relative and absolute ages can now be compared by obtaining estimates for the values of δ and $\varepsilon_E/\dot{D}_{int,2}$.

It should be noted that though we are comparing the specific case of two vessels, in general ε_E can be thought of as the variation of $\dot{D}_{ext}$ within a studied ceramic assemblage, and $\dot{D}_{int,2}$ can be considered as an order of magnitude estimate of $\dot{D}_{int}$, and δ is an estimate of the typical ratio between $\dot{D}_{ext}$ and $\dot{D}_{int}$. Thus, if we denote the mean of the internal dose rate measurements as μ_I, the mean of the external dose rate measurements as μ_E, and the uncertainty of the external dose rate measurements as σ_E, we may write:

$$\delta \sim \frac{\mu_E}{\mu_I} \tag{25}$$

and

$$\frac{1}{R}\frac{\varepsilon_E}{\dot{D}_{int,2}}\left(1-\delta\right) \sim \frac{1}{R}\frac{\sigma_E}{\mu_I}\left(1-\delta\right). \tag{26}$$

The quantity μ_I is easily calculated since the internal dose rates in this analysis are assumed to be known. The value of R will of course vary with each vessel. In contrast, since the external dose rates for the vessels in question are assumed to be unknown, some additional source of information will be necessary to determine μ_E and σ_E. Since these are only required to estimate the error, and do not affect the age calculation itself, order-of-magnitude approximations will be sufficient: a set of values taken from a similar assemblage, or surveys of the region, may be used (e.g. following Zink et al., 2012).

As an example, we consider the first application of this methodology in Part B of this paper (Hood et al., 2019). As no measurements for $\dot{D}_{ext}$ existed for the vessels under consideration, μ_E and σ_E were estimated using existing values from the literature and from measurements taken from material at a different site (of similar age and composition). This was justifiable, because the values for $\dot{D}_{ext}$ measurements across a wide geographical region around the site were very similar to one another. In this work the actual values were $\sigma_E \sim 0.108$, $\mu_I \sim 1.61$, $\mu_E \sim 0.726$, and $R \sim 1$ on average, meaning that the relative error estimates were

$$\frac{1}{R}\frac{\sigma_E}{\mu_I}\left(1-\delta\right) \sim 3.6\%, \tag{27}$$

for the relative ages and

$$\delta \sim \frac{\mu_E}{\mu_I} \sim 45\% \tag{28}$$

for the absolute ages.

In summary, this section shows that the relative age of two vessels is obtained (equation 19) by dividing the D_e of one vessel by that of another, and then dividing by the associated ratio of the $\dot{D}_{int}$ measurements for each vessel; effectively, this is calculating the ratio of the two absolute ages while neglecting $\dot{D}_{ext}$. Furthermore, it demonstrates that when neglecting $\dot{D}_{ext}$ the relative error in the *absolute* age for a given vessel is ∼45%, but the relative error in the *relative* age for a given vessel, is only ∼3.6% (an error which is small when added in quadrature to the relative error of the equivalent dose measurement).

4. How to calculate a relative age sequence

In Section 2, we derived a formula for the relative age, R, of two vessels. We now lay out briefly a program for calculating the relative ages of a group of vessels (whose external dose rates satisfy the conditions given in Section 1).

1. For each of the vessels, determine the equivalent dose (D_e) and the internal dose rate ($\dot{D}_{int}$) in the usual way.

2. Select a vessel to be used as a reference vessel. This vessel may be selected for a number of reasons, for example:

   (a) Low uncertainty on its D_e and $\dot{D}_{int}$ measurements (which will reduce the uncertainty across the other relative ages).

   (b) The vessel has a known absolute age, e.g. through a known $\dot{D}_{ext}$, or by associated radiocarbon or historical chronologies.

   (c) The vessel has an age that is central to the sequence.

3. Calculate the relative age of every vessel in the sequence. If we define the equivalent dose of our selected reference vessel to be $D_{e,ref}$ and the internal dose rate of the reference vessel to be $\dot{D}_{int,ref}$, then for all the other vessels, the relative age R of the vessel may be calculated as follows:

$$R = \frac{D_e}{D_{e,ref}}\frac{\dot{D}_{int,ref}}{\dot{D}_{int}} \tag{29}$$

4. The uncertainty in the relative age is composed of two parts: the error that comes from neglecting $\dot{D}_{ext}$, given in equation (26), and the error that comes from uncertainties in $\dot{D}_{int}$ and D_e. These can be combined in quadrature, as they are uncorrelated with each other. Thus if ε_R is defined to be the absolute uncertainty in the relative age, and ε_D the absolute uncertainty in the equivalent dose, we may write:

$$\frac{\varepsilon_R}{R} \sim \sqrt{\left(\frac{\varepsilon_I}{\dot{D}_{int}}\right)^2 + \left(\frac{\varepsilon_{I,ref}}{\dot{D}_{int,ref}}\right)^2 + \left(\frac{\varepsilon_D}{D_e}\right)^2 + \left(\frac{\varepsilon_{D,ref}}{D_{e,ref}}\right)^2 + \left(\frac{1}{R}\frac{\sigma_E}{\mu_I}\left(1-\delta\right)\right)^2} \tag{30}$$

where σ_E, the estimated variation in the external dose rate, and μ_I, the average internal dose rate, are defined in the previous section.

5. Discussion

Luminescence dating can be used as a relative dating method to establish a relative chronology for an archaeological assemblage. This paper has outlined the mathematical formula with which to calculate this age, and the error associated with determining the sequence.

Using OSL dating as a relative dating tool would be most beneficial for work on museum collections, or in any case where the original contextual information for an assemblage is lacking. Additionally, it would be adventageous in detecting forgeries.

It may be the case that some parts of the external dose rate, for example the cosmic dose rate ($\dot{D}_{cos}$), are known, and indeed that there may be other more unusual external doses, coming from, for example, a storage location since excavation, x-ray imaging, CT-scanning, and so on. In this case, the above analysis may be simply adapted as follows: all known doses should be included in the calculation of $\dot{D}_{int,1}$, $\dot{D}_{int,2}$, and of course μ_I, with μ_E and σ_E being the estimated mean and uncertainty of the remaining unknown dose received by each vessel.

Finally, once a relative sequence has been calculated, if one member of the sequence has an associated absolute date calculated by other means (i.e. radiocarbon dating), the whole sequence can then be anchored and the absolute ages of all the vessels can be derived (within error bars). This powerful result seems somewhat counterintuitive; however, it is merely a result, principally, of assuming that the variation in the external dose rate is small compared to the size of the internal dose rate (a condition often true for pottery), and holds as long as this is the case (it should be noted, additionally, that any errors in the single absolute date will apply systematically to the whole sequence).

6. Conclusion

In summary, this paper has provided a framework for implementing luminescence dating as a relative dating method. While the usefulness of this technique will be heavily dependent upon individual assemblages and the quality of available relative dating methods, further potential for this technique is significant in the museum world and further advances in the study of archaeological assemblages can be made as a result.

Acknowledgments

This research was completed owing to the generous support of the G.A. Wainwright Fund; the Clarendon Fund; Merton College, Oxford; and Brasenose College, Oxford.

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Reviewer

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